

Ashay (TM)

$ax^2 + bx + c = 0$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + 2\alpha + 3\alpha = 12$

$6\alpha = 12$   
 $\alpha = 2$

Richard

$\sum \alpha^2 = \frac{-b}{a} = \frac{12}{1} = 12$

$6p = 12$   
 $p = 2$

$\sum \alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$

$\alpha\beta\gamma = \frac{-d}{a} = \frac{48}{1} = 48$

$(2 \times 4) + (2 \times 6) + (4 \times 6) = 48$

$a = 44$

$6 \times 4 = 24$

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METHOD #3

1 (a) The quadratic equation  $x^2 - 2x + 4 = 0$  has roots  $\alpha$  and  $\beta$ .

(i) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . [2]

(ii) Show that  $\alpha^2 + \beta^2 = -4$ . [2]

(iii) Hence find a quadratic equation which has roots  $\alpha^2$  and  $\beta^2$ . [3]

(b) The cubic equation  $x^3 - 12x^2 + ax - 48 = 0$  has roots  $p$ ,  $2p$  and  $3p$ .

(i) Find the value of  $p$ . [2]

(ii) Hence find the value of  $a$ . [2]

(Q8, June 2005)

$ax^2 + bx + c$

$(x - \alpha^2)(x - \beta^2) = 0$

$x^2 - x\beta^2 - x\alpha^2 + \alpha\beta^2$

$x^2 - (\alpha^2 + \beta^2)x + (\alpha\beta)^2$   
 $x^2 + 4x + 16$

$\sum \alpha^2 \beta^2 = \sum (\alpha\beta)^2$

$\sum \alpha^2 = -4 = \frac{-b}{a}$      $c = \frac{c}{a}$     METHOD #2

$a = 1$      $b = 4$      $c = 4$

$y^2 + 4y + c = \frac{c}{a} = \frac{16}{1} = 16$

$y^2 + 4y + 16$

~~$(x-p)(x-2p)(x-3p)$   
 $= (x^2 - 3px + 2p^2)(x-3p)$~~

Using this method we could do i) and iii)

MASCO

i)  $\alpha + \beta = \frac{-2}{1} = -2$

$\alpha\beta = 4$

ii)  $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$(-2)^2 = \alpha^2 + 8 + \beta^2$

$4 = \alpha^2 + 8 + \beta^2$

$\alpha^2 + \beta^2 + 8 = 4$

$\alpha^2 + \beta^2 = -4$

iii)  $y = x^2$     METHOD #1

$\sqrt{y} = x$

$(\sqrt{y})^2 - (2 \times \sqrt{y}) + 4 = 0$

$y - 2\sqrt{y} + 4 = 0$

~~$y^2 + 4y + 16 = 0$~~

Marcio!

Ashey

Richard

$$\int_2^3 y^2 - 6y + 9$$

$$= \left[ \frac{y^3}{3} - 3y^2 + 9y \right]_2^3$$

$$= \left[ \frac{3^3}{3} - (3 \times 3^2) + (9 \times 3) \right] - \left[ \frac{2^3}{3} - (3 \times 2^2) + (9 \times 2) \right]$$

$$= [9] - \left[ \frac{26}{3} \right]$$

$$= \frac{1}{3}$$

$$\int_0^2 \frac{y}{2}$$

$$= \frac{1}{2} \left[ \frac{1}{4} y^2 \right]$$

$$= \frac{1}{4} \cdot 4 - 0$$

$$= 1$$

$$x = \pm \sqrt{\frac{y}{2}}$$

$$V = \pi \left[ \int_2^3 (-y+3)^2 + \int_0^2 \frac{y}{2} \right]$$

$$= \frac{4\pi}{3}$$

3. Fig. 6 shows the region enclosed by part of the curve  $y = 2x^2$ , the straight line  $x + y = 3$ , and the y-axis. The curve and the straight line meet at P(1, 2).

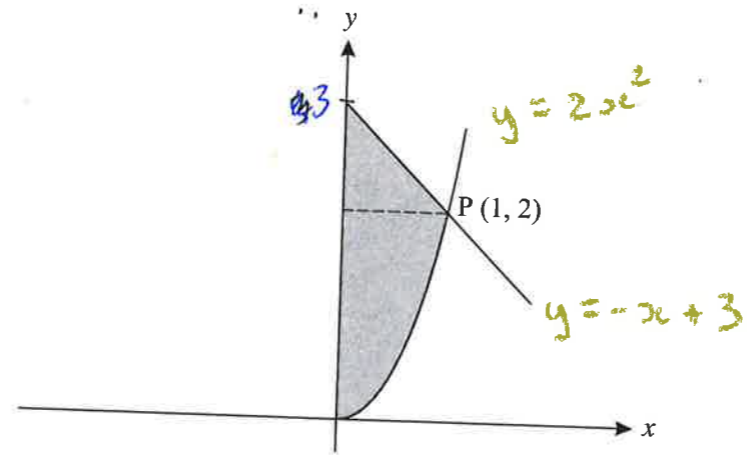


Fig. 6

The shaded region is rotated through  $360^\circ$  about the y-axis. Find, in terms of  $\pi$ , the volume of the solid of revolution formed. [7]

$$x = \sqrt{\frac{y}{2}}$$

$$x = -y + 3$$

$$-x + 3 = 0$$

$$x = 3$$

$$y = 3$$

when revolved:



+



=



$$\sum \alpha\beta = b$$

$$(16 - 30i)(16 + 30i) = b$$

$$(i) \quad x = 16 + 30i$$

$$\begin{aligned} & (x - 16 - 30i)(x - 16 + 30i) \\ &= x^2 - 16x + \cancel{30xi} - 16x + 256 + \cancel{-480i} \\ &= \cancel{x^2} - 32x - \cancel{30ix} + 480i + 900 \\ &= x^2 - 32x + 1156 \end{aligned}$$

$$a = -32 \quad b = 1156$$

$$\begin{aligned} z &: 16 + 30i \\ \bar{z} &: (x - 16 + 30i)(x - 16 - 30i) \\ &= x^2 - 16x - 30ix - 16x + 16^2 - 480i + 900 \\ &= x^2 - 32x + 1156 \end{aligned}$$

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19 One root of the quadratic equation  $x^2 + ax + b = 0$ , where  $a$  and  $b$  are real, is  $16 - 30i$ .

(i) Write down the other root of the quadratic equation.

[1]

(ii) Find the values of  $a$  and  $b$ .

[4]

(Q9, June 2011)