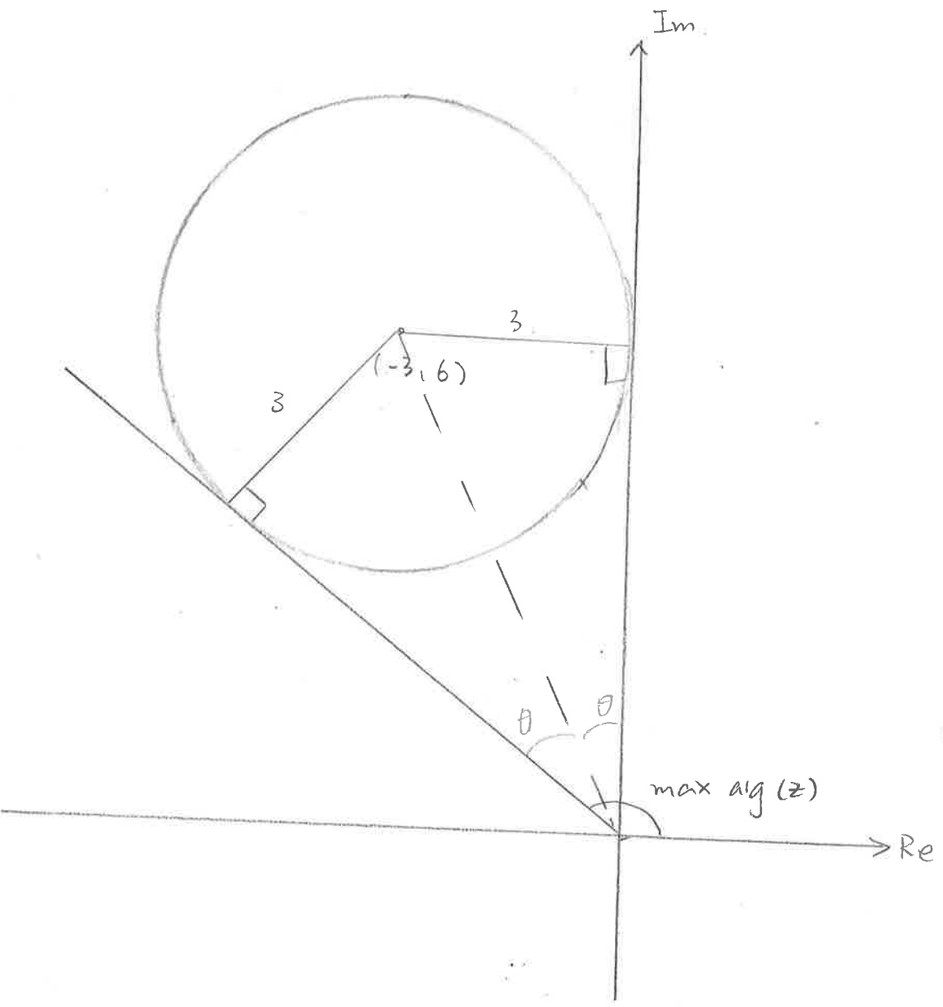
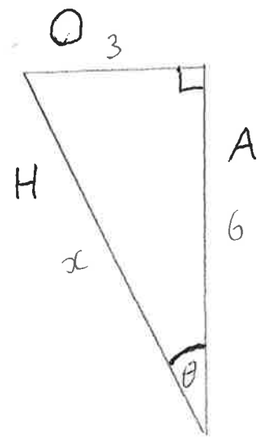
16 The complex number z satisfies $|z + 3 - 6i| = 3$. Show that the exact maximum value of $\arg z$ in the interval $(-\pi, \pi)$ is $\frac{\pi}{2} + 2\arcsin\left(\frac{1}{\sqrt{5}}\right)$. **(4 marks)**





$x = \sqrt{6^2 + 3^2}$

$= 3\sqrt{5}$

$\theta = \sin^{-1}\left(\frac{3}{3\sqrt{5}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\max \arg(z) = 2\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \frac{\pi}{2}$

Emre, Nanya

Q1.

(a) Shade on an Argand diagram the set of points

$$\left\{ z \in \mathbb{C} : |z-4\mathrm{i}| \leqslant 3 \right\} \cap \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z+3-4\mathrm{i}) \leqslant \frac{\pi}{4} \right\}$$

(6)

The complex number w satisfies

$$|w-4\mathrm{i}| = 3$$

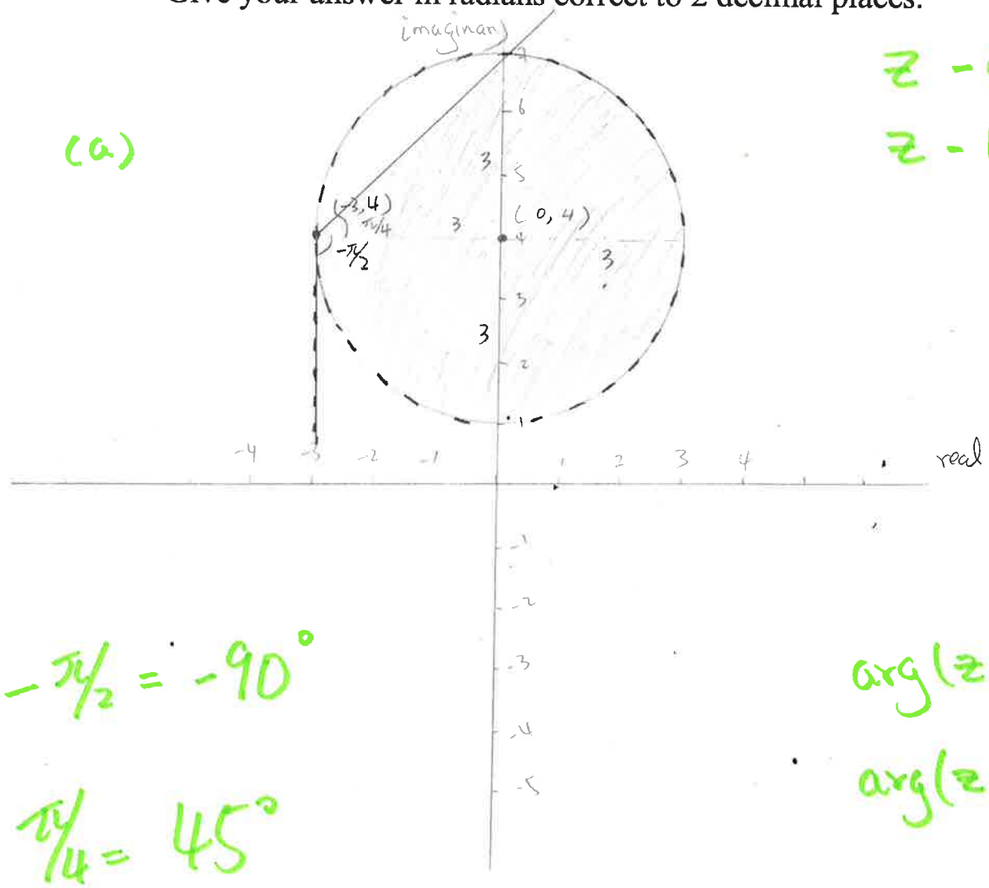
(b) Find the maximum value of arg w in the interval $(-\pi, \pi]$.

Give your answer in radians correct to 2 decimal places.

(2)

(Total for question = 8 marks)

(a)



$z - 4i$

$z - (4i)$

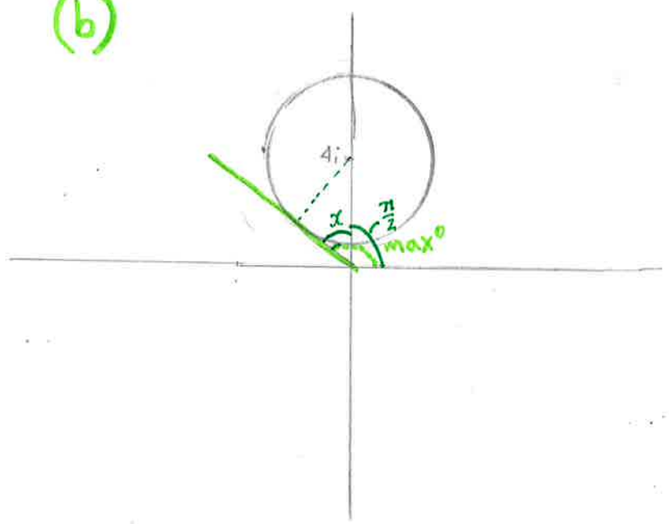
$-\pi/2 = -90°$

$\pi/4 = 45°$

$\arg(z+3-4i)$

$\arg(z-(-3+4i))$

(b)



$\sin(x) = \frac{3}{4}$

$x = \sin^{-1}\left(\frac{3}{4}\right)$

Max Value for arg w

$= \sin^{-1}\left(\frac{3}{4}\right) + \frac{\pi}{2} = 2.42°$

Ananya Leming :-

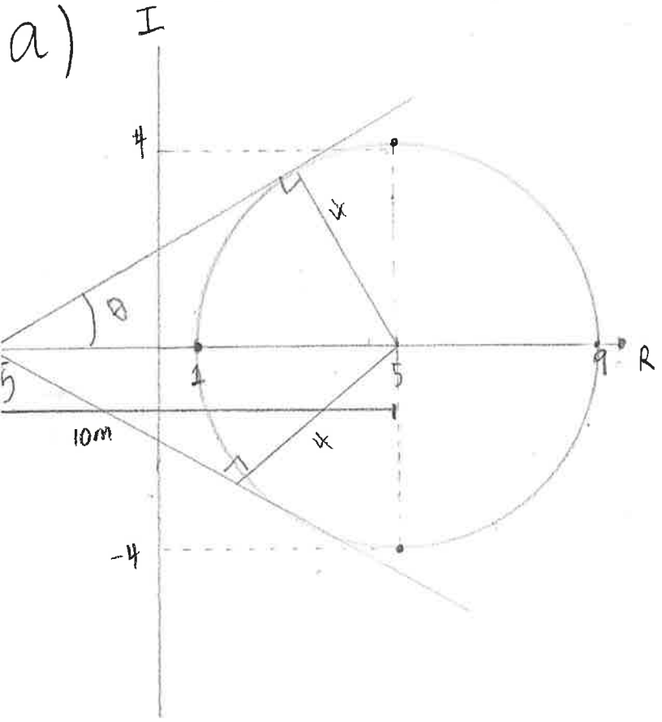
17 A complex number z is represented by the point P on the Argand diagram. Given that $|z - 5| = 4$,

a sketch the locus of P. **(2 marks)**

b Find the complex numbers that satisfy both $|z - 5| = 4$ and $\arg(z + 3i) = \frac{\pi}{3}$, giving your answers in radians to 2 decimal places. **(6 marks)**

c Given that $\arg(z + 5) = \theta$ and $|z - 5| = 4$ have no common solutions, find the range of possible values of θ, $-\pi < \theta < \pi$. **(3 marks)**

a)



B) $|z - 5| = 4$

$|x - 5 + yi| = 4$

$(x-5)^2 + y^2 = 16$

$\arg(z + 3i) = \frac{\pi}{3}$

$\arg(x + 3i + yi) = \frac{\pi}{3}$

$\frac{y+3}{x} = \tan\left(\frac{\pi}{3}\right)$

$y + 3 = \sqrt{3}x$

$y = \sqrt{3}x - 3$

$(x-5)^2 + (\sqrt{3}x - 3)^2 = 16$

$4x^2 + (-10 - 6\sqrt{3})x + 18 = 0$

$x_1 = 3.962 \qquad x_2 = 1.135$

$y_1 = 3.86 \qquad y_2 = -1.03$

$\therefore$

$z_1 = 3.96 + 3.86i$

$z_2 = 1.14 - 1.03i$

C) $\sin^{-1}\left(\frac{4}{10}\right) = 0.41^c$ (2sf)

$0.41^c < \theta < \pi$

$-\pi < \theta < -0.41^c$

Makar & Lydia